

Equilibrium without Commitment

Suppose the government cannot commit to policy A announced at beginning of time : $\{\tau_1^A, \tau_2^A, \delta^A, B^A\}$.

Instead, at the beginning of period 2, the government can change $\{\tau_2^A, \delta^A\}$ to a **New** policy for period 2, $\{\tau_2^N, \delta^N\}$.

This setting makes the Ramsey outcome unsustainable. Suppose the government's announced plan is the Ramsey outcome. So $\delta^A > 0$ (and $\tau_2^A > 0$); committing to repaying the international creditors. But if a benevolent government can reconsider its period 2 policy at the beginning of $t=2$, they will set $\delta^N = 0$ and $\tau_2^N = 0$. This reduces the $t=2$ labor tax on the household to 0 since the government doesn't repay the international creditors. Clearly this makes the household best off in $t=2$ as they can consume $\delta^A B^A$ more. So the government reneges on its original commitment to pay back the creditors.

This leads us to the idea of a sustainable equilibrium. In a sustainable equilibrium, we allow a benevolent government to lack commitment; however, in equilibrium, their chosen policy in each period is the same as their original policy announced at the beginning of time. Furthermore, the household sticks with its original plan throughout time as well. Thus, period by period, the government (and household) optimally sticks to its original plan.

In a sustainable equilibrium in this environment, we need best response functions for the household for both periods: (f_1, f_2) . These are called "allocation rule" of the household.

For any $\tilde{\pi}_1 = \{\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\delta}, \tilde{B}\}$, $f_1(\tilde{\pi}_1)$ is the household's optimal allocation at $t=1$ given $\tilde{\pi}_1$:
 $f_1(\tilde{\pi}_1) = \{c_1^*(\tilde{\pi}_1), l_1^*(\tilde{\pi}_1), c_2^*(\tilde{\pi}_1), l_2^*(\tilde{\pi}_1)\}$

For any $\tilde{\pi}_2 = \{\tilde{\tau}_2, \tilde{\delta}\}$, $f_2(\tilde{\pi}_2)$ chooses the household's optimal allocation at $t=2$ given $\tilde{\pi}_2$:
 $f_2(\tilde{\pi}_2) = \{c_2^*(\tilde{\pi}_2), l_2^*(\tilde{\pi}_2)\}$.

We also need a policy mapping σ , which is a function that takes $\tilde{\pi}_1 = \{\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\delta}, \tilde{B}\}$ as an input and outputs a $t=2$ policy.

In the sustainable equilibrium, this mapping will tell us the best choice of period 2 taxes and default rate, given the original $t=1$ policy plan and the household's best response in $t=2$, f_2 :

$$\sigma(\tilde{\pi}_1) = \{\tau_2^*(\tilde{\pi}_1), \delta^*(\tilde{\pi}_1)\}.$$

Now, let's define the sustainable equilibrium in this setting.

A sustainable equilibrium consists of:

i) Government policy: $\hat{\pi}_1 = \{\hat{\tau}_1, \hat{\tau}_2, \hat{\delta}, \hat{\beta}\}$

ii) Policy mapping: σ

iii) HH allocation rules: f_1, f_2

iv) Price for bond: $\hat{q}$, such that

1) For all $\tilde{\pi}_1$, $f_1(\tilde{\pi}_1) = \{c_1^*(\tilde{\pi}_1), l_1^*(\tilde{\pi}_1), c_2^*(\tilde{\pi}_1), l_2^*(\tilde{\pi}_1)\}$ solves HH's problem in $t=1$:

$$f_1(\tilde{\pi}_1) = \arg \max_{c_1, l_1, c_2, l_2} U(c_1, c_2, l_1, l_2)$$

s.t. $c_1 \leq (1 - \tilde{\tau}_1) l_1$
 $c_2 \leq (1 - \tilde{\tau}_2) l_2$
 $l_1, l_2 \in [0, 1]$; non-neg

2) For all $\tilde{\pi}_2$, $f_2(\tilde{\pi}_2) = \{c_2^*(\tilde{\pi}_2), l_2^*(\tilde{\pi}_2)\}$ solves the HH's problem in $t=2$.

$$f_2(\tilde{\pi}_2) = \arg \max_{c_2, l_2} \log(c_2) + \Psi \log(1 - l_2)$$

s.t. $c_2 \leq (1 - \tilde{\tau}_2) l_2$
 $l_2 \in [0, 1]$; $c_2 \geq 0$

Note: (1) + (2) just define the HH's best response functions f_1, f_2 .

3) Given $\hat{\pi}_1 = \{ \hat{\tau}_1, \hat{\tau}_2, \hat{\delta}, \hat{\beta} \}$ and σ ,
 f_2 chooses the continuation of
 $f_1(\hat{\pi}_1) = \{ \hat{c}_1, \hat{l}_1, \hat{c}_2, \hat{l}_2 \}$ in period $t=2$;
 i.e. $f_2(\sigma(\hat{\pi}_1)) = \{ \hat{c}_2, \hat{l}_2 \}$

4) Given f_1 , $\hat{\pi}_1 = \{ \hat{\tau}_1, \hat{\tau}_2, \hat{\delta}, \hat{\beta} \}$ solves:

$$\begin{aligned} \tilde{\pi}_1 = \{ \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\delta}, \tilde{\beta} \} \quad & \max_{\tilde{\pi}_1} U(c_1^*(\tilde{\pi}_1), l_1^*(\tilde{\pi}_1), c_2^*(\tilde{\pi}_1), l_2^*(\tilde{\pi}_1)) \\ \text{s.t.} \quad & q = \tilde{\tau}_1 l_1^*(\tilde{\pi}_1) + \hat{q} \tilde{\beta} \\ & \tilde{\delta} \tilde{\beta} = \tilde{\tau}_2 l_2^*(\tilde{\pi}_1) \end{aligned} \quad \leftarrow \text{Note } \hat{q}.$$

5) Given f_2 , for all $\tilde{\pi}_1 = \{ \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\delta}, \tilde{\beta} \}$

$\sigma(\tilde{\pi}_1) = \{ \tau_2^*(\tilde{\pi}_1), \delta^*(\tilde{\pi}_1) \}$ solves:

$$\begin{aligned} \bar{\pi}_2 = \{ \bar{\tau}_2, \bar{\delta} \} \quad & \max_{\bar{\pi}_2} \log(c_2^*(\bar{\pi}_2)) + \psi \log(1 - l_2^*(\bar{\pi}_2)) \\ \text{s.t.} \quad & \bar{\delta} \bar{\beta} = \bar{\tau}_2 l_2^*(\bar{\pi}_2) \end{aligned}$$

6) Given $\hat{\pi}_1 = \{ \hat{\tau}_1, \hat{\tau}_2, \hat{\delta}, \hat{\beta} \}$, σ chooses
 the continuation value of $\hat{\pi}_1$ in period $t=2$;

i.e. $\sigma(\hat{\pi}_1) = \{ \hat{\tau}_2, \hat{\delta} \}$

7) Given $\hat{\pi}_1$ and σ , B solves international creditor's problem.

8) Markets clear:

$$\hat{c}_1 + g = \hat{l}_1 + \hat{q} \hat{B}$$

$$\hat{c}_2 + \hat{\delta} \hat{B} = \hat{l}_2$$

This completes the definition.

As shown in the notes, the unique sustainable equilibrium for this setting is:

- $\hat{\pi}_1 = \{\hat{c}_1, \hat{c}_2, \hat{\delta}, \hat{B}\} = \{(1+\psi)g, 0, 0, 0\}$
- $\sigma(\hat{\pi}_1) = \{c_2^*(\hat{\pi}_1), \delta^*(\hat{\pi}_1)\} = \{0, 0\} \forall \hat{\pi}_1$
- $\hat{q} = 0$
- f_1, f_2 can be solved for using $t=1$ and $t=2$ HH problem

Intuitively, the international creditors would never buy debt from a government that would always choose to default; so $\hat{q} = 0$. Since $\hat{q} = 0$, the government will optimally choose $\hat{B} = 0$. Thus g is funded solely through $t=1$ labor taxes. As $\hat{\delta} = 0$, the Ramsey outcome is not sustainable.

[Note: To see that $q=0$ means the gov will not sell its debt i.e. $B=0$, notice that qB is what gov receives, and pays back δB].