

Macroeconomic Theory (ECON 8107)

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Midterm Exam

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Please write down the answers in the provided booklet. There are a total of 3 questions. You have 2 hours to work on this exam.

1. **(40 points)** Consider the following endowment economy setup. The measure of households is one. A household i gets endowment y_{it} at time t . Every household's endowments follow an m -state first order **Markov process** over time with a stationary transition probability $\pi(y'|y) = \text{Prob}(y_{t+1} = y' | y_t = y)$. The Markov process can take on values in the set $Y = \{y_1, \dots, y_m\}$, where all values are strictly positive. Assume law of large numbers hold and that the initial distribution of endowments were drawn from the invariant distribution Π of the Markov process. Thus there is no aggregate uncertainty. Households' preferences are given by

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

The utility function u is bounded, strictly increasing, strictly concave and continuously differentiable. Further assume u satisfies Inada. The discount factor satisfies $\beta \in (0, 1)$. The only asset is the one-period risk-free bond which has price q_t . The household's period t budget constraint is $c_t + q_t b_{t+1} \leq b_t + y_t$. The borrowing constraint is such that $b_{t+1} \geq \psi$. For parts (a)-(c), we consider a stationary setup with $q_t = q > \beta \forall t$.

- a) (5 pts) Using b_t and y_t as the state variables, write down the Bellman equation for a household's problem.
- b) (5 pts) Characterize the savings and consumption decisions of the household. Make sure you include the case where the borrowing constraint binds.
- c) (10 pts) Let $b' = g_b(b, y; q)$ and $c = g_c(b, y; q)$ be the optimal policy functions governing the savings and consumption decisions of the household. Show that g_b is non-decreasing in b (for each y).
- d) (10 pts) Define a recursive competitive equilibrium for this model.
- e) (5 pts) Provide an outline of how you would adapt your definition in (d) to write a *stationary* competitive equilibrium.
- f) (5 pts) Given the current setup, explain whether or not we could also write this problem recursively with wealth $a_t = b_t + y_t$ as the **only** state variable. If we cannot, provide conditions such that we could and write the Bellman equation.

2. **(25 points)** An unemployed worker receives each period an offer to work for wage w_t forever; where $w_t = w$ in the first period of work, and $w_t = \phi^t w$ after t periods on the job. Assume $\phi > 1$, that is, wages increase with tenure. The initial wage offer is drawn from distribution $F(w)$ that is constant over time (entry-level wages are stationary); successive drawings across periods are independently and identically distributed from the support of the distribution $[0, B]$. The worker's objective function is to maximize $\mathbb{E}[\sum_{t=0}^{\infty} \beta^t y_t]$. If the worker is employed, $y_t = w_t$, and if the worker is unemployed $y_t = c \in (0, B)$, where c is unemployment compensation. Let $v(w)$ be the optimal value of the objective function for an unemployed worker who has w in hand. Additionally, assume $\phi\beta < 1$.

- (a) (5 pts) Write the Bellman equation for the problem of an unemployed worker with current wage offer w .
- (b) (10 pts) Prove that the optimal policy of the worker takes the form of a reservation wage $w^* \in [0, B]$. (You may use without proof that the Bellman equation is a contraction mapping as long as you state *all* conditions needed.)
- (c) (10 pts) Provide an equation for the reservation wage, w^* . Suppose that there are two economies, $i \in \{A, B\}$ that only differ in the growth rate of wages of employed workers; $\phi_A > \phi_B$. Show that economy A has the lower reservation wage, i.e. $w_A^* < w_B^*$. Assume $\phi_i\beta < 1 \ \forall i$.

3. **(35 points)** Consider a setting with two agents $i \in \{1, 2\}$, each of measure 1. There is one good in each period and history, $c_t^i(s^t)$. The agents can invest in capital and supply labor in the market, and are endowed with one unit of time each period each (which can be split into leisure and labor). Agents of each type own capital and can rent it to the firm. Both agent types have the same discount factor $\beta \in (0, 1)$. The utility function of each household is strictly concave, strictly increasing, differentiable, and satisfies Inada conditions. Also assume the period utility function of for agent i is $u^i(c_t^i(s^t), l_t^i(s^t))$. We will refer to $l_t^i(s^t)$ as labor supplied by agent i , so that $l_t^i(s^t) \in [0, 1]$. The firm in the economy produces an output good with a constant return to scale technology that uses capital and labor, but is subject to shocks to aggregate productivity $A(s_t)$, where s_t is assumed to follow a first order Markov process that takes on a finite number of values in S . Thus total output is $A(s_t)F(K_t, L_t)$ (where K_t, L_t are the aggregate capital and labor). Assume full depreciation of capital, $\delta = 1$.

- (a) (15 pts) Define a recursive competitive equilibrium with Arrow securities.

Now assume that there is a representative household with preferences of the form: $\mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t u(c_t, c_{t-1})]$ The time endowment is 1 and the production technology and stochastic process is the same as above.

- (b) (5 pts) Write the social planner's problem as a dynamic programming problem.
- (c) (15 points) Define a recursive competitive equilibrium with a decentralized household problem. You do not need to include Arrow Securities.¹

¹Hint: Think hard about the state variables.