

# Island Model - Different Timing

I will consider a different timing than in other notes.

Timing on an island with workforce  $x$  and shock  $z$ :

| Start period                          | Middle Period                                            | End Period                                                                                                | Start new period ...                      |
|---------------------------------------|----------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|-------------------------------------------|
| $(x, z)$<br>↓<br>Determines $f(x, z)$ | Workers move<br>to or from island.<br>Workforce now $x'$ | People who stayed<br>or immigrated here<br>earn wage $f(x, z)$ .<br>People who moved<br>away earn wage 0. | $(x', z')$<br>↓<br>Determines $f(x', z')$ |

Immigrants can move to islands in the current period and earn the wage at  $(x, z)$ . So I assume that inflows or outflows does not affect the current wage  $f(x, z)$ , which is fixed by the beginning of period workforce  $x$  and shock  $z$ .

I'm not sure if this is the assumption Chari wants.

I assume immigrants who chose to leave their island yesterday and today move to the best alternative island earn a fixed value of  $\theta$ .  $\theta$  is the present value of earnings from today onward from moving to the best alternative island.

Island type analysis given  $(x, z)$ :

Island 1: Positive Flow of Immigrants in Current Period

This occurs if:

$$\theta < f(x, z) + \beta \sum_{z' | z} v(x, z') \pi(z', z)$$

Positive flow of immigrants ( $x' > x$ ) will occur until:

$$\theta = f(x, z) + \beta \sum v(x', z') \pi(z', z)$$

Thus the Bellman equation for workers on this island type is:

$$V(x, z) = \theta$$

So the value function for workers on this island type is constant  $\forall x, z$ . So all islands of this type have the same "value," which is  $\theta$ .

Island 2: All workers leave ↗ Not sure about "a")"

This occurs if:

$$f(x, z) + \beta \sum_{z'|z} v(x, z') \pi(z', z) < \beta \theta$$

Workers who move today earn zero wage today but are promised present value wages valuing  $\theta$  in the following period; thus its current value is  $\beta \theta$ .

Workers leave ( $x' < x$ ) until:

$$f(x, z) + \beta \sum_{z'|z} v(x', z') \pi(z', z) = \beta \theta$$

So Bellman equation for workers in this island is:

$$v(x, z) = \beta \theta$$

Island 3: No inflows nor outflows;  $x = x'$

This occurs if:

$$\beta \theta < f(x, z) + \beta \sum_{z'|z} v(x, z') \pi(z', z) < \theta$$

As  $x = x'$ , the Bellman equation is:

$$v(x, z) = f(x, z) + \beta \sum_{z'|z} v(x, z') \pi(z', z)$$

Now, combining these into common Bellman.

If move today, get value  $\beta\theta$  (0 wage today + discounted value of moving to best island tomorrow).

If stay, either on an island with positive inflows or with no inflows/outflows.

Positive inflows condition:

$$\theta < \underbrace{f(x, z) + \beta \sum_{z'|z} v(x, z') \pi(z', z)}$$

No inflows/outflows condition:

$$f(x, z) + \beta \sum_{z'|z} v(x, z') \pi(z', z) < \theta$$

Use min in value function.

and  $\beta\theta < \}$

Thus,

$$v(x, z) = \max \left\{ \underbrace{\beta\theta}_{\text{leave}}, \underbrace{\min \left\{ \theta, f(x, z) + \beta \sum_{z'|z} v(x, z') \pi(z', z) \right\}}_{\text{stay}} \right\}$$