

1) a) Define a CE:

A CE consists of:

1) HH alloc: $\hat{Z}^H \equiv \{C_{1t}, C_{2t}, l_t, M_{t+1}\}_{t=0}^{\infty}, M_0$ given

2) Firm alloc: $\hat{Z}^F \equiv \{y_t^f, l_t^f\}_{t=0}^{\infty}$

3) Gov Policy: $G \equiv \{M_t^G, T_t\}_{t=0}^{\infty}$

4) Prices: $P \equiv \{p_t, w_t\}_{t=0}^{\infty}$, s.t.

i) Given G and P , HH chooses $\hat{Z}^H$ to solve:

$$\max_{\hat{Z}^H} U(\cdot)$$

$$\text{s.t. } M_{t+1} + p_t C_{1t} + p_t C_{2t} \leq M_t + w_t l_t + T_t \quad \forall t$$

$$p_t C_{1t} \leq M_t \quad \forall t$$

non-neg

ii) Given G and P , firm chooses $\hat{Z}^F$ to solve:

$$\max \sum_{t=0}^{\infty} p_t y_t^f - w_t l_t^f$$

$$\text{s.t. } y_t^f \leq l_t^f$$

non-neg

iii) Gov's budget balances

$$M_{t+1}^b = M_t^b + T_t \quad \forall t$$

iv) Markets clear:

$$M_t^b = M_t \quad \forall t$$

$$C_{1t} + C_{2t} = y_t^f = l_t^f \quad \forall t$$

b) CIA binds. $M_t = \bar{M} \quad \forall t \geq 0$

Firm: $w_t = p_t$

HH FOCs: λ_t on t BC; θ_t on t CIA.

$$[C_{1t}]: \frac{B^t}{C_{1t}} = p_t [\lambda_t + \theta_t] = p_t \lambda_{t-1} \quad [\text{by } M_t \text{ FOC}]$$

$$[C_{2t}]: \frac{\alpha B^t}{C_{2t}} = p_t \lambda_t \quad \left. \vphantom{\frac{\alpha B^t}{C_{2t}}} \right\} \alpha(1-l_t) = \psi C_{2t}$$

$$[l_t]: \frac{B^t \psi}{1-l_t} = \frac{w_t}{p_t} \lambda_t$$

$$[M_{t+1}]: \lambda_t = \lambda_{t+1} + \theta_{t+1}$$

$$\text{From CIA: } p_t C_{1t} = \bar{M} \quad \forall t \Rightarrow \frac{p_t C_{1t}}{p_{t-1} C_{1,t-1}} = 1$$

$$C_{1t} \text{ FOC: } \beta^t = \underbrace{C_{1t} p_t}_{=} \lambda_{t-1}$$

$$\Rightarrow \beta^t = \underbrace{C_{1t-1} p_{t-1}}_{=} \lambda_{t-1} \text{ from CIA.}$$

$$\Rightarrow \frac{\beta^t}{C_{1t-1}} = p_{t-1} \lambda_{t-1} = \frac{\alpha \beta^{t-1}}{C_{2t-1}}$$

$$\Rightarrow \frac{\beta}{\alpha} C_{2t-1} = C_{1t-1}$$

$$\Rightarrow C_{1t} = \frac{\beta}{\alpha} C_{2t}$$

We have .

$$C_{1t} = \frac{\beta}{\alpha} C_{2t}$$

$$1 - l_t = \frac{\psi}{\alpha} C_{2t} \Rightarrow l_t = 1 - \frac{\psi}{\alpha} C_{2t}$$

$$+ C_{1t} + C_{2t} = l_t$$

$$\Rightarrow C_{2t} \left(\frac{\beta}{\alpha} + 1 \right) = 1 - \frac{\psi}{\alpha} C_{2t}$$

$$\Rightarrow C_{2t} \underbrace{\left(1 + \frac{\beta}{\alpha} + \frac{\psi}{\alpha} \right)}_{\sigma} = 1$$

$$C_{2t} = \frac{1}{\sigma} = \frac{\alpha}{\alpha + \beta + \psi}$$

$$y_t^f = l_t = 1 - \frac{\psi}{\alpha} \frac{1}{\sigma} = 1 - \frac{\psi}{\alpha + \beta + \psi} = \frac{\alpha + \beta}{\alpha + \beta + \psi}$$

$$C_{1t} = \frac{\beta}{\alpha \sigma} = \frac{\beta}{\alpha + \beta + \psi}$$

$$p_t = p_{t+1} \quad \forall t \Rightarrow p_0 = \frac{\bar{M}}{\bar{C}_0}$$

} $\forall t$

$$\bar{C}_{1t} = \frac{\beta}{\alpha + \beta + \psi}$$

$$\bar{C}_{2t} = \frac{\alpha}{\alpha + \beta + \psi}$$

$$\bar{y}_t^f = \bar{l}_t = \frac{\alpha + \beta}{\alpha + \beta + \psi}$$

$$p_t = \frac{\bar{M}}{\bar{C}_0}$$

Economy is stationary.

$$c) \quad \begin{array}{l} M_0 = M_1 = \bar{M} \\ M_t = \hat{M} \quad t \geq 2 \end{array} ; \quad \hat{M} > \bar{M}$$

For $t=0$ & $t \geq 2$ onwards, have same allocations as in (b). Can apply same logic as above.

$$\text{I.e.: } t=0 : p_0 C_{10} = \bar{M} = p_1 C_{11}$$

$$\text{FOC: } \frac{\beta}{C_{11}} = p_1 (\lambda_1 + \theta_1) = p_1 \lambda_0$$

$$\Rightarrow p_1 C_{11} = \frac{\beta}{\lambda_0} = \bar{M} = p_0 C_{10}$$

$$\Rightarrow \frac{\beta}{C_{10}} = p_0 \lambda_0 = \frac{\alpha}{C_{20}} \Rightarrow C_{10} = \frac{\beta}{\alpha} C_{20}$$

$$\text{and } \frac{\psi}{1-l_0} = \frac{\alpha}{C_{20}} \Rightarrow 1-l_0 = \frac{\psi}{\alpha} C_{20} \Rightarrow l_0 = 1 - \frac{\psi}{\alpha} C_{20}$$

$$\Rightarrow \text{Plug into } t=0 \text{ MC: } c_{20} \left(\frac{\beta}{\alpha} + 1 \right) = 1 - \frac{\psi}{\alpha} c_{20}$$

$$\Rightarrow c_{20} = \frac{\alpha}{\alpha + \beta + \psi} \quad \text{and } c_{10}, l_0 \text{ follow.}$$

Same logic for $t=2$.

Only need $t=1$:

$$\text{CIA: } \frac{p_2 c_{12}}{p_1 c_{11}} = \frac{\hat{M}}{\bar{M}} = \mu > 1$$

$$\Rightarrow p_2 c_{12} = \mu p_1 c_{11}$$

$$\text{FOC } c_{12}: \beta^2 = p_2 c_{12} \lambda_1 = \mu p_1 c_{11} \lambda_1$$

$$\frac{\beta^2}{\mu c_{11}} = p_1 \lambda_1 = \frac{\alpha \beta^1}{c_{21}}$$

$$\Rightarrow c_{11} = \frac{\beta}{\alpha \mu} c_{21} \quad \left. \vphantom{\frac{\beta}{\alpha \mu} c_{21}} \right\} + \text{MC}$$

$$l_1 = 1 - \frac{\psi}{\alpha} c_{21}$$

$$\Rightarrow c_{21} \left(1 + \frac{\beta}{\alpha \mu} + \frac{\psi}{\alpha} \right) = 1$$

$$\Rightarrow \hat{c}_{21} = \frac{\alpha \mu}{\alpha \mu + \beta + \mu \psi}; \quad \hat{c}_{11} = \frac{\beta}{\alpha \mu + \beta + \mu \psi}$$

$$= \hat{l}_1 = y_1 = 1 - \frac{\psi \mu}{\alpha \mu + \beta + \mu \psi} = \frac{\alpha \mu + \beta}{\alpha \mu + \beta + \mu \psi}$$

$$\Rightarrow \hat{l}_1 = \frac{\alpha M + \beta}{\mu(\alpha + \psi) + \beta} \stackrel{?}{<} \frac{\alpha + \beta}{\alpha + \psi + \beta} = \bar{l}_1 \quad \text{from (b)}$$

$$\begin{aligned} & (\alpha + \beta)(\mu(\alpha + \psi) + \beta) \stackrel{?}{>} (\alpha M + \beta)(\alpha + \psi + \beta) \\ &= \cancel{\alpha \mu(\alpha + \psi)} + \alpha \beta + \beta \mu(\alpha + \psi) + \beta^2 \\ & \stackrel{?}{>} \cancel{\alpha \mu(\alpha + \psi)} + \alpha \mu \beta + \beta(\alpha + \psi) + \beta^2 \end{aligned}$$

$$\Rightarrow \cancel{\alpha \beta} + \cancel{\beta \mu \alpha} + \beta \mu \psi > \cancel{\alpha \mu \beta} + \cancel{\beta \alpha} + \beta \psi$$

$$\Rightarrow \beta \psi(\mu - 1) > 0 \quad \checkmark \quad \text{as } \mu > 1.$$

We see that $\hat{l}_1$ is lower than $\bar{l}_1$.

$$\text{But, } \hat{l}_t = \bar{l}_t \quad \forall t \neq 1.$$

$$\text{Similarly, } \hat{c}_{11} < \bar{c}_{11} \quad \text{but } \hat{c}_{1t} = \bar{c}_{1t} \quad \forall t \neq 1$$

$$\text{and } \hat{c}_{21} > \bar{c}_{21} \quad \text{but } \hat{c}_{2t} = \bar{c}_{2t} \quad \forall t \neq 1.$$

The next page shows how the values change over time in this economy.

$$\hat{C}_{1t} = \begin{array}{ccc} t=0 & & t=1 \\ \frac{\beta}{\alpha + \beta + \psi} & \xrightarrow{\text{Dec}} & \frac{\beta}{m\alpha + \beta + m\psi} \end{array} \xrightarrow{\text{Inc}} \begin{array}{ccc} t=2 & & \\ \frac{\beta}{\alpha + \beta + \psi} & \xrightarrow{\text{constant}} & \end{array}$$

$$\hat{C}_{2t} = \begin{array}{ccc} \frac{\alpha}{\alpha + \beta + \psi} & \xrightarrow{\text{Inc}} & \frac{\alpha m}{m\alpha + \beta + m\psi} \end{array} \xrightarrow{\text{Dec}} \begin{array}{ccc} \frac{\alpha}{\alpha + \beta + \psi} & \xrightarrow{\text{constant}} & \end{array}$$

$$\hat{y}_t^f = \hat{l}_t = \begin{array}{ccc} \frac{\alpha + \beta}{\alpha + \beta + \psi} & \xrightarrow{\text{Dec}} & \frac{\alpha m + \beta}{m\alpha + \beta + m\psi} \end{array} \xrightarrow{\text{Inc}} \begin{array}{ccc} \frac{\alpha + \beta}{\alpha + \beta + \psi} & \xrightarrow{\text{constant}} & \end{array}$$

$$\hat{p}_t = \begin{array}{ccc} \frac{\bar{M}}{\hat{C}_{10}} & \xrightarrow{\text{Inc}} & \frac{\bar{M}}{\hat{C}_{11}} \end{array} \xrightarrow{\text{Inc}} \begin{array}{ccc} \frac{\hat{M}}{\hat{C}_{10}} & \xrightarrow{\text{constant}} & \end{array}$$

d)/c) Bonds in BC. If $R_t > 1$ CIA binds
BC now:

$$M_{t+1} + B_{t+1} + p_t C_{1t} + p_t C_{2t} = M_t + R_t B_t + w_t l_t + T_t$$

FOCs:

$$[C_{1t}]: \frac{\beta^t}{C_{1t}} = p_t [\lambda_t + \theta_t]$$

$$[C_{2t}]: \frac{\frac{C_{1t}}{\alpha} \beta^t}{C_{2t}} = p_t \lambda_t$$

$$[M_{t+1}]: \lambda_t = \lambda_{t+1} + \theta_{t+1}$$

$$[B_{t+1}]: \lambda_t = \lambda_{t+1} R_{t+1}$$

$$\text{Comp. slackness: } \left\{ \begin{array}{l} \theta_t (M_t - p_t C_{1t}) = 0 \\ \theta_t \geq 0 ; \quad M_t - p_t C_{1t} \geq 0 \end{array} \right.$$

Clearly $\lambda_{t+1} > 0$ from FOCs

$$\text{From } B_{t+1} \text{ FOC: } \lambda_t = \lambda_{t+1} R_{t+1} > \lambda_{t+1} \text{ since } R_{t+1} > 1$$

$$\Rightarrow \lambda_t > \lambda_{t+1}$$

$$\Rightarrow \lambda_t - \lambda_{t+1} > 0$$

$$\text{From } M_{t+1} \text{ FOC: } \theta_{t+1} = \lambda_t - \lambda_{t+1} > 0$$

If $\theta_t > 0 \Rightarrow M_t - p_t C_{1t} = 0$
by complementary slackness.