

Macroeconomic Theory (ECON 8107)

Final Exam

March 6th, 2023

Please write down the answers in the provided booklet. There are a total of 3 questions. You have 2 hours and 20 minutes to work on this exam.

1. (30pts) Consider a cash-credit goods economy in which an infinitely lived household maximizes its discounted lifetime utility given by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [\log(c_{1t}) + \alpha \log(c_{2t}) + \psi \log(1 - l_t)]$$

where c_1 denotes the consumption of cash goods, c_2 denotes the consumption of credit goods, and l is the supply of labor. Cash and credit goods are produced using the following technology in each period $t \geq 0$:

$$c_{1t} + c_{2t} = l_t$$

At the beginning of each period, fiat currency is traded in a securities market. The household's securities market constraint is

$$M_t \leq (M_{t-1} - p_{t-1}c_{1,t-1}) - p_{t-1}c_{2,t-1} + w_{t-1}l_{t-1} + T_{t-1}$$

where M_t denotes cash balances, p_t denotes the price level and T_t denotes lump-sum transfers by the government. The cash in advance constraint is

$$p_t c_{1t} \leq M_t.$$

At the end of a period, worker is paid for its labor supply, meets with the shopper, credit balances are settled, and household enters next period's security market. Each household starts with an initial holding of cash M_0 .

- a) (5pts) Define a competitive equilibrium.
- b) (10pts) Suppose the cash in advance constraint binds in each period. Assume that monetary policy is described by a sequence of constant money supplies, $M_t = \bar{M}$ for all t . Solve for the set of competitive equilibria.
- c) (10pts) Suppose the cash in advance constraint binds in each period. Assume now that the government introduced an expansionary monetary policy of the form:

$$\begin{aligned} M_0 &= M_1 = \bar{M} \\ M_t &= \hat{M} \quad \forall t \geq 2 \end{aligned}$$

where $\hat{M} > \bar{M}$. Solve for set of competitive equilibria. Compare c_{1t}, c_{2t}, l_t, y_t and p_t for periods $t = 0, 1, 2$ from the allocations obtained in (b). Show that output falls in response to an anticipated monetary expansion.

- d) (5 pts) Just for this subquestion, assume that there are also bonds in the budget constraint. Prove that if $R_t > 1$, then the cash-in-advance constraint binds.
2. (35pts) Consider a two period ($t = 1, 2$) small open economy; there is a perishable consumption good in the economy which is produced by a constant returns to scale technology that transforms one unit of labor into one unit of consumption good. The utility of a representative household in this economy is given by

$$u(c_1, c_2, l_1, l_2) = \log(c_1) + \psi \log(1 - l_1) + \beta [\log(c_2) + \psi \log(1 - l_2)]$$

The household is endowed with one unit of labor. Assume that $0 < \psi < \beta < 1$. We will assume the consumer does not have access to asset markets. There is a benevolent government that has to finance an exogenous amount of expenditures in the first period, given by $g > 0$. To do so, government has access to a proportional tax rate on labor income in both periods, τ_1 and τ_2 , and an international financial market. The international financial market works as the following: Government can sell its debt B in this market at unit price q in the first period to risk neutral creditors, who discount future at rate β . The government can decide with what probability it is going to default on its debt in the second period, $(1 - \delta)$, and has to run a balanced budget in each period (you can think of δ as the fraction of its debt that the government is going to pay back in period $t = 2$).

- a) (5pts) Define a Ramsey equilibrium in an environment where the government can commit.
- b) (10pts) Fully characterize the Ramsey outcome.
- c) (10pts) Assume that the government lacks commitment. Define a sustainable equilibrium.
- d) (10pts) Show that the Ramsey outcome is not sustainable.
3. (30pts) Consider the following economy with a representative infinitely lived consumer, a continuum of monopolistic intermediate good producers (of unit mass), competitive final good producers, and a government; in each period t , the economy is subject to an exogenous i.i.d shock that takes values in the set S , according to the probability measure π . Let $s^t \in S^t$ denote the history of shocks up to date t . Final good producers have access to the following technology to transform intermediate goods to final consumption good:

$$Y_t(s^t) = \left[\int_0^1 y_t(i, s^t)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

The price index of the consumption good is given by

$$P_t(s^t) = \left[\int_0^1 p_t(i, s^t)^{(1-\epsilon)} di \right]^{\frac{1}{1-\epsilon}}$$

Each intermediate good producer is a monopolist that produces with a cost function Ψ (and marginal cost function ψ) and obtains the demand for its product from the final good's producer. Moreover, they set prices in Calvo-fairly fashion. In particular, in each period t after the shock s_t is realized, these producers have a probability of $1 - \theta$ of being able to set a new price. A consumer is endowed with one unit of labor. His or her preferences over streams of consumption, labor, and money are represented by the following expected utility form that satisfies the usual concavity and Inada requirements:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) [u(c_t(s^t), l_t(s^t)) + v(M_{t+1}(s^t)/p_t(s^t))]$$

and a budget constraint of the form

$$p_t(s^t)c_t(s^t) + M_{t+1}(s^t) + Q_t(s^t)B_{t+1}(s^t) \leq w_t(s^t)l_t(s^t) + M_t(s^{t-1}) + T_t(s^t) + B_t(s^{t-1}) + \Pi(s^t)$$

where $M_{t+1}(s^t)$ denotes the holdings of nominal money balances, $B_t(s^t)$ is government debt, $Q_t(s^t)$ is the price of the **one period risk free bond**, $p_t(s^t)$ is the price level, $\Pi_t(s^t)$ is the household's entitlement to nominal profits of the firms, and $T_t(s^t)$ are the lump-sum transfers.

- a) (5pts) State the final good firm's profit maximization problem and find the demand for intermediate goods.
- b) (5pts) Write the problem of the consumer.
- c) (5pts) From the first order conditions of the consumer, derive (i) the Euler Equation, (ii) the intratemporal equation for labor and consumption, and (iii) the equation relating money demand, consumption and the price of the bond.
- d) (5pts) Define $q_{t,t+k}(s^{t+k}|s^t)\pi(s^{t+k}|s^t)$ the value of one unit of consumption in state s_{t+k} in terms of consumption in state s^t . Write down the problem the intermediate firm solves in every t and s^t .
- e) (5pts) Find the optimal pricing policy for the intermediate good producers.
- f) (5pts) Call the optimal price in e) p_t^* . Find the expression for inflation in this economy. (*Hint: First find the aggregate price index in t as a function of p_t^* and last period's aggregate price (remember that a fraction $1-\theta$ of firms are allowed to update their prices each period.)*)