

Macroeconomic Theory (Econ 8106)

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Final Exam

There are a total of 2 questions. You have 1 hour and 30 minutes to work on this exam and send the solution by **6 pm**. Good luck!

Question 1: Endogenous Growth and Taxation (60 points)

Consider the following Planner's Problem:

$$\begin{aligned} & \max_{\{(c_t, k_{t+1}, h_{t+1}, x_{ht}, x_{kt})\}} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{s.t. } & c_t + x_{ht} + x_{kt} \leq Ak_t^\alpha h_t^{1-\alpha}, \\ & k_{t+1} \leq x_{kt} + (1 - \delta_k)k_t, \\ & h_{t+1} \leq x_{ht} + (1 - \delta_h)h_t, \\ & c_t, k_{t+1}, h_{t+1}, x_{ht}, x_{kt} \geq 0, \\ & k_0 \text{ and } h_0 \text{ given} \end{aligned}$$

Suppose the government has a fixed sequence of expenditures, $\{g_t\}_{t=0}^\infty$, it must finance through taxes on capital and labor income, τ_{kt} and τ_{ht} .

- (a) (5 pts) Define a Tax Distorted Competitive Equilibrium (TDCE) for this economy in an Arrow-Debreu setup, where government is allowed to borrow and lend (one lifetime budget constraint).
- (b) (10 pts) Write down the system of equations that characterize the TDCE.
- (c) (15 pts) Derive the Implementability Constraint.
- (d) (5 pts) Write down the Ramsey Problem for a benevolent government.
- (e) (10 pts) Assume $\delta_k = \delta_h$. What should $\frac{\tau_{kt}}{\tau_{ht}}$ be according to the Ramsey Problem?
- (f) (15 pts) Assume $\delta_k = \delta_h$. Show that $\lim_{t \rightarrow \infty} \tau_{kt} = 0$ and $\lim_{t \rightarrow \infty} \tau_{ht} = 0$ according to the Ramsey problem. Given taxes are zero in the long-run, how is government funding its expenditure g_t which may be non-zero at steady-state?

Question 2: Labor Augmenting Technological Change(40 points)

Consider the following neoclassical growth model: there is a representative agent born with k_0 units of initial capital stock and 1 unit of labor. Capital stock depreciates at rate δ and representative consumer's utility is

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \text{ where } \sigma > 0.$$

Firms produce output by a production function, $F(k_t, A_t n_t)$ where labor-augmenting technological progress $A_t = \gamma^t$. So, planner's problem can be written as follows:

$$\begin{aligned} & \max_{\{(c_t, n_t, x_t, k_t)\}} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{s.t. } & c_t + x_t \leq F(k_t, \gamma^t n_t) \\ & k_{t+1} \leq x_t + (1 - \delta)k_t, \\ & n_t \leq 1, \\ & c_t, n_t, x_t, k_t \geq 0 \\ & k_0 \text{ given} \end{aligned}$$

- (a) (15 points) Impose minimal assumptions required to write the planner's problem for this economy as a *stationary dynamic program*, and write the planner's problem in the recursive form(Bellman Equation).
- (b) (25 points) Use your assumptions in part (a) to prove this economy converges to balanced growth path for any given level of initial capital stock.

Question 2 Capital Taxation and SPP (40 points)

Consider an infinite horizon setting in which there is a representative agent and a representative firm as in the standard single sector growth model. The utility function of the representative consumer is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where u has the usual properties. The firm has a concave CRS production function. Total resources in the economy must satisfy

$$c_t + x_t + g_t \leq F(k_t, n_t)$$

In order to finance government spending, the government taxes purchases of investment goods in period t at rate τ_{xt} . It has to balance its budget **every** period. All investment is made at the household level and the law of motion is

$$k_{t+1} \leq x_t + (1 - \delta)k_t$$

SHOW that the equilibrium allocation of this economy solves a planner's problem. Make sure to explicitly state the SPP. Remember that the SPP should not directly include prices or taxes.

Why is it useful for us to be able to rewrite the problem as a SPP ?