

Macroeconomic Theory (Econ 8106)

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Problem Set 3

Due 12/5 at midnight

Question 1: TDCE (Prelim SP2006, Q. I.1)

- (a) Define a general Taxed Distorted Competitive Equilibrium for the one-sector growth model with multiple households and multiple firms.
- (b) Show that if an equilibrium allocation and price system satisfy:
 - The feasibility constraint for each firm holds with equality
 - The budget constraint for each household holds with equality
 - All markets for all goods clear with equality

Then the Government Budget constraint is automatically satisfied. Make sure you clearly state any assumptions you need to make.

Question 2: Equivalent Taxes (Prelim SP2005, Q. I.3)

Consider a dynamic, representative agent, one sector economy in which the government must finance a given stream of expenditures, $\{g_t\}_{t=0}^{\infty}$, using only consumption and labor income taxes, τ_{ct} and τ_{nt} .

- (a) Define a Tax Distorted Competitive Equilibrium (TDCE) for this setting.
- (b) Show that any allocation resulting in an equilibrium of this sort can also be realized as an equilibrium in a world where the government must finance the same sequence of expenditures $\{g_t\}_{t=0}^{\infty}$ but can only use labor and capital income taxes, $\hat{\tau}_{kt}$ and $\hat{\tau}_{nt}$.
- (c) Is $\hat{\tau}_{nt} = \tau_{nt}$? That is, for a given sequence of government expenditures, and a given equilibrium allocation with that sequence in the two types of tax systems described above, are the two required time series of labor income taxes the same?

Question 3: Capital Taxation with Lump Sum Rebates (Prelim FA2013, Q. II)

Consider an infinite horizon, representative consumer, economy in which preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where c_t is consumption in period t . Each household has an initial stock of capital given by k_0 and is subject to the law of motion for capital $k_{t+1} \leq x_t$, where x_t is investment by household in period t . Assume that labor is inelastically supplied with 1 unit each period. Output is given by

$$c_t + x_t \leq F(k_t, l_t)$$

where F satisfies all of the normal conditions.

Assume that government taxes income earned on capital at rate τ in every period and that all revenue generated in this way is lump sum rebated to the households. There are no other taxes, and no other government spending.

- (a) Carefully define a Tax Distorted Competitive Equilibrium for this environment, given the fiscal policy.
- (b) Show that the allocation that results from the TDCE you defined in Part (a) can also be obtained by solving a Planning Problem with a representative consumer. Clearly and carefully lay out what this Planning Problem is, and show that its solution is the TDCE allocation from Part (a).

Now assume that τ is a tax on all income earned by the consumer each period and that government revenue is used to purchase government consumption. Thus, output is given by,

$$c_t + x_t + g_t \leq F(k_t, l_t)$$

- (c) Define a Tax Distorted Competitive Equilibrium for this environment, given the fiscal policy.
- (d) Show that the allocation that results from the TDCE you defined in Part (c) can also be obtained by solving a Planning Problem with a representative consumer. Clearly and carefully lay out what this Planning Problem is, and show that its solution is the TDCE allocation from Part (c).

Question 4: Ramsey's Problem & Optimal Taxation (Prelim SP2014 Q.II)

Consider an infinite horizon setting in which there is a representative consumer and a representative firm as in the standard single sector growth model. The utility function of the representative consumer is given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

The feasibility constraint for the firm is:

$$c_t + x_t + g_t \leq F(k_t, n_t)$$

where g_t is the amount of government goods and services produced in each period.

There is no technological change. Investment is done at household level, and the standard law of motion for capital is assumed to hold.

Suppose that government has at its disposal only on labor income and consumption taxes for financing expenditures (i.e. no capital income taxes), but can freely borrow and lend (i.e., it faces a present value budget constraint). Assume that the consumer takes $g_t, \tau_{nt}, \tau_{ct}$ as given when making decisions.

- (a) Set up and define a TDCE (Tax Distorted Competitive Equilibrium) for each fixed sequence $\{(g_t, \tau_{nt}, \tau_{ct})\}_{t=0}^{\infty}$.
- (b) Assume interior solution of the equilibrium, give the FOC's that characterize the equilibrium you defined in Part (a).
- (c) What is the Implementability Constraint for this economy? Compare this to what you would get if the government could choose labor and capital income taxes, but not consumption taxes.
- (d) If g_t is a fixed sequence and the government acts benevolently in choosing (τ_{nt}, τ_{ct}) , will it be true that $\tau_{ct} \rightarrow 0$? Does $\tau_{nt} \rightarrow 0$? If not, what can you say about $\lim_{t \rightarrow \infty} \tau_{ct}$ and $\lim_{t \rightarrow \infty} \tau_{nt}$?

(Assume that in the Ramsey allocation, all quantities converge to constant levels, $c_t \rightarrow c_{\infty}$, etc.)

Question 5: Sequential and present value budget constraints

Consider an economy with an infinite horizon representative consumer and representative firm with constant returns to scale technology producing both, the consumption good and the investment good.

- (a) Assume that the government taxes consumption expenditures, labor income and capital income in order to balance its sequential budget constraint each period (i.e. one-period budget constraint with borrowing and lending of government bonds d_t^g):

$$(1 + r_t)d_t^g + g_t \leq \tau_{nt}\tilde{w}_t n_t + \tau_{kt}\tilde{r}_t^k k_t + \tau_{ct}c_t + d_{t+1}^g \quad \forall t \geq 0$$

Define a TDCE in this environment. Note that the representative consumer will face a sequential budget constraint as well.

- (b) Assume that the government taxes consumption expenditures, labor income and capital income in order to balance the present value budget constraint:

$$\sum_{t=0}^{\infty} p_t g_t \leq \sum_{t=0}^{\infty} \tau_{nt} w_t n_t + \tau_{kt} r_t k_t + \tau_{ct} p_t c_t$$

Define a TDCE in this environment. Note that the representative consumer will face a present value (Arrow-Debreu) budget constraint as well.

- (c) State and prove a result showing that the real equilibrium allocations arising from these TDCEs are equivalent. In your proof, state clearly the amount of borrowing and lending in economy (a) as a function of objects in the (b) economy. Also make sure you show the relationship between prices in the two equilibria.

Hint: This problem is closely related to the ADE/SME equivalence results we covered in 8105.

Question 6: Steady-State for a model with taxes (Prelim 2020)

Consider an infinite horizon, single sector, representative agent model with no technological change, and inelastic labor supply. Assume that the government uses a constant tax, τ , applied to all (i.e., both labor and capital) income uniformly. Assume that the proceeds of this tax are used, balancing the budget in every period, to purchase a stream of goods, g_t . Make the standard assumptions on preferences and production functions.

- (a) Define a TDCE (Tax Distorted Competitive Equilibrium) for this economy.
- (b) Characterize the steady-state of this TDCE as a function of τ . Say as much as you can about what happens to k_{ss} , c_{ss} and g_{ss} as a function of τ .
- (c) Show that no matter what the initial capital stock is, and no matter what the τ is, the equilibrium path of the economy converges to the steady-state you derived in part 2.