

Macroeconomic Theory (Econ 8106)
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Problem Set 1

Question 1

Create graphs for each of the time series below and cite your data sources. Under each graph, provide a description of the series, including any noticeable trends and how the series varies with the business cycle (is it countercyclical or procyclical and why). Additionally, create a table that contains the averages and standard deviations of each of the series below.

- (a) Consumption as a fraction of GNP
- (b) Investment as a fraction of GNP
- (c) Government spending as a fraction of GNP
- (d) Payments to labor as a share of GNP
- (e) Payments to capital as a share of GNP
- (f) The capital-to-output ratio
- (g) The growth rate of output per capita
- (h) The growth rate of consumption per capita
- (i) The rate of return of capital
- (j) PCE inflation rate

Question 2

Detrend the following series for the United States. Report the trends and residuals, as well as their statistical properties. Provide a description of the filter you used, including the mathematical equations underlying it. Explain why detrending time series data can be useful for macroeconomists. The Hodrick-Prescott filter (mentioned in class) is one of the options for detrending.¹

- (a) Output per capita
- (b) Consumption per capita
- (c) Investment per capita

¹However, see James Hamilton's paper "Why You Should Never Use the Hodrick-Prescott"

- (d) Hours worked per capita
- (e) Unemployment rate

Question 3

Consider the competitive equilibrium of an economy with an “ Ak ” production function $F(k) = Ak$, where $A > 0$. Note that there is no labor choice nor are there wages in this economy. There are two types of agents with an equal mass of each, but they have different discount factors: $\beta_1 > \beta_2$. Note that $\beta_i \in (0, 1) \ \forall i$. The utility function of type $i \in \{1, 2\}$ is given by:

$$\sum_{t=0}^{\infty} \beta_i^t \log(c_t^i)$$

a. Define and characterize an Arrow-Debreu equilibrium for this economy. Make sure you include the Euler equations and transversality conditions in your characterization.

b. Prove the following statement:

$$\lim_{t \rightarrow \infty} \frac{\hat{c}_t^2}{\hat{c}_t^1} = 0$$

Given that $\beta_1 > \beta_2$, explain whether or not this result makes sense.

Question 4

The following questions deal with a 2 sector economy with firms producing consumption goods indexed by $j_c = 1, 2, \dots, J_c$ and firms producing investment goods indexed by $j_x = 1, 2, \dots, J_x$. Assume CRS technology for each firm in each sector. There are n consumers indexed by $i = 1, 2, \dots, n$ and preferences $U^i(\underline{c}, \underline{l}) = \sum_{t=0}^{\infty} \beta^t u^i(c_t^i, l_t^i)$, where $0 < \beta < 1$.

- (a) Define an Arrow-Debreu equilibrium.
- (b) Write down the necessary first-order conditions that a solution to each consumer’s problem must satisfy.
- (c) Suppose the solution to the consumer’s problem is interior, i.e. all quantities at all times are strictly positive. Derive a condition relating the price of investment goods and the rental rate of capital. State briefly in words what this means.
- (d) Write down the necessary first-order conditions that a solution to each firm’s problem must satisfy. Derive a condition relating the price of consumption goods and the price of investment goods.
- (e) Prove that, in equilibrium, consumers cannot make positive profits by investing.
Hint: Show that in equilibrium, the constraints of the consumer’s problem can be re-written so that only the initial endowment of capital enters the budget constraint. You will also use the transversality condition for agent i .
- (f) Define a sequential markets equilibrium.

- (g) Read section 4 of Professor Jones' notes. Describe how and when we can use the firm aggregation results presented in this section.
- (h) Can this problem be solved as a social planner's problem? Explain the two different methods (representative consumer approach and Negishi method).
- (i) Assume that all preferences and firms are not identical (so we can't use the aggregation results). Write the social planner's problem. Make sure you state any additional assumptions you need for the problem to be solved as a social planner's problem.
- (j) Assume that preferences, endowments, and firms are identical. Write the social planner's problem using the representative household approach (from section 4 of Professor Jones' notes). Make sure you state any additional assumptions you need for this problem to be solved using this representative household approach.
- (k) Assume that preferences are identical and homothetic, endowments are potentially different across agents, and firms are identical. Write the social planner's problem using a representative household (from section 4 of Professor Jones' notes). Make sure you state any additional assumptions you need for this problem to be solved using this representative household approach.
- (l) Suppose you have obtained the equilibrium allocation for the social planner's problem. Explain how can you find the allocations and prices for the Arrow-Debreu equilibrium in the n person economy in the following cases. Make sure you state the required conditions on utility functions and production functions for each case.
 - (a) Allow agents to have different preferences and endowments, and firms to be different. **Hint:** Think back to the Negishi method.
 - (b) All firms, preferences, and endowments are identical.
 - (c) All firms are identical, preferences are identical and homothetic, but endowments are different across agents.

Question 5

For this problem, please read the introductions to chapters 2 and 4 as well as sections 2.1 and 4.1 (also 4.2 to get a head start on next week) of Stokey, Lucas, and Prescott's (SLP) *Recursive Methods in Economic Dynamics*. It is available online through the university library. This problem focuses on the section 4.1 of SLP (pages 67-77). This section is very important as it tells us when the social planner's problem and Bellman equation are equivalent; i.e. when the Principle of Optimality is satisfied.

Suppose we have a single infinitely lived agent with preferences:

$$\sum_{t=0}^{\infty} \beta^t c_t^\rho$$

where $\beta \in (0, 1)$ and $\rho \in (0, 1)$. The production function is Cobb-Douglas (with labor inelastically supplied at 1):

$$F(k, 1) = \theta k_t^\alpha$$

where $\theta > 0$ and $\alpha \in (0, 1)$. The law of motion of capital is:

$$k_{t+1} \leq i_t + (1 - \delta)k_t$$

Where $\delta \in (0, 1)$. Therefore, the feasibility constraint is:

$$c_t + k_{t+1} - (1 - \delta)k_t \leq \theta k_t^\alpha$$

As always, consumption and capital stock must be non-negative.

- (a) Write down the social planner's problem in this environment.
- (b) Translate the social planner's problem into SLP's sequence problem (SP) form (page 66).
 - What is X in this environment? How about x_t and x_{t+1} ?
 - What is the period return function F in this case? What is it a function of? How do we obtain it?
 - What is the feasibility correspondence Γ in this case? What is its input?
- (c) Write down the Bellman equation for this problem. In each period, what is the state variable and what are the control variables? Explain the difference between state and control variables in general.
- (d) Translate the Bellman equation into SLP's functional equation (FE) form (page 66).

Next, we want to verify that the “Principle of Optimality” (page 67) holds.

- (e) In your own words, describe the idea behind the Principle of Optimality.
- (f) State Assumption 4.1. In your own words, explain what it means. Explain why it is satisfied in our current environment (Hint: what value of k' can always be chosen from $\Gamma(k)$?).
- (g) State Assumption 4.2. In your own words, explain what it means. Also explain why it is satisfied in our current environment (Hint: think back to why we discussed the maximum sustainable capital).

Theorems 4.2 and 4.3 relate v^* from the SP and v from FE.

- (h) State Theorem 4.2. Since assumptions 4.1 and 4.2 are satisfied, we know this theorem holds in our current environment. Explain what “ v^* satisfies (FE)” means in your own words.²

² v^* is formally defined on page 70.

- (i) State Theorem 4.3. Why is this only a “partial converse” to Theorem 4.2?

Theorems 4.4 and 4.5 relate optimal plans $\{x_t^*\}_{t=0}^\infty$ from the SP and FE.

- (j) State Theorem 4.4. Explain what it means in your own words.
- (k) State Theorem 4.5. Explain how we know that condition (11) holds in this case (Hint: Given that F is bounded above, what is the highest possible value of v^* ?).
- (l) Define an optimal policy correspondence. What did we call the “optimal policy correspondence” in 8105?
- (m) Write the last sentence of section 4.1 in your own words and adapt it to our current environment.

You have shown that the SP from our current environment is essentially equivalent to the recursive formulation (i.e. Bellman equation or FE). You showed that the optimum and optimal policies of the SP and FE coincide for any initial $k_0 \in K$. In other words, the Principle of Optimality is satisfied. Given that the Bellman equation is much easier to solve than the social planner’s problem, this is a pretty powerful result! However, we still do not know if the FE v is unique nor do we know any of its properties. We also don’t know anything about the properties of the policy correspondence. This is all covered in Section 4.2 (next week!).