

Macro Theory (8106)

Fall 2022

Final Exam

Please write down the answers in the provided booklet. There are a total of 2 required questions and one Extra Credit question. You have 1 hour and 30 minutes to work on this exam.

Question 1: Endogenous Growth and Taxation (60 points)

Consider the following Planner's problem

$$\max_{\{c_t, x_{ht}, x_{kt}, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to:

$$c_t + x_{ht} + x_{kt} \leq Ak_t^\alpha h_t^{1-\alpha} \quad \forall t \geq 0$$
$$k_{t+1} \leq k_t(1 - \delta_k) + x_{kt} \quad \forall t \geq 0$$
$$h_{t+1} \leq h_t(1 - \delta_h) + x_{ht} \quad \forall t \geq 0$$
$$c_t, k_{t+1}, h_{t+1} \geq 0 \quad \forall t \geq 0$$
$$k_0, h_0 \text{ given}$$

Suppose the government has a fixed sequence of expenditure $\{g_t\}_{t=0}^{\infty}$ that it has to finance through taxes on capital τ_{kt} and labour τ_{ht}

- (a) (5pts) Define a Tax Distorted Competitive Equilibrium (TDCE) for this economy in an Arrow-Debreu setup, where the government is allowed to freely “borrow and lend” (one lifetime/present value budget constraint).
- (b) (10pts) Write down the system of equations that characterize the TDCE.
- (c) (15pts) Derive the Implementability Constraint.
- (d) (5pts) Write down the Ramsey Problem for a benevolent government.
- (e) (10pts) Assume $\delta_h = \delta_k$. What should $\frac{\tau_{kt}}{\tau_{ht}}$ be according to the Ramsey Problem?
- (f) (10pts) Assume $\delta_h = \delta_k$. Show that $\lim_{t \rightarrow \infty} \tau_{kt} = 0$ and $\lim_{t \rightarrow \infty} \tau_{ht} = 0$.
- (g) (5pts) How does this compare to the standard Chamley-Judd result (for both τ_{kt} and τ_{nt})? Give some intuition. Additionally, given that taxes are zero in the long-run, how is government funding its expenditure g_t which may be non-zero at steady-state?

Question 2: Taxation and SPP (40 points)

Consider an infinite horizon setting in which there is a representative agent and a representative firm as in the standard single sector growth model. The utility function of the representative consumer is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where u has the usual properties. The consumer is endowed with 1 unit of time each period, which can be divided between labor n_t and leisure l_t (so that $n_t + l_t \leq 1$ for all t). The firm has a concave CRS production function. Total resources in the economy must satisfy

$$c_t + x_t + g_t \leq F(k_t, n_t)$$

In order to finance government spending on fixed $\{g_t\}_{t=0}^{\infty}$, the government taxes:

- i.) Purchases of investment goods in period t at rate τ_{xt} **and**
- ii.) Purchases of consumption goods in period t at a rate τ_{ct} **and**
- iii.) Capital and wage income in period t at a rate of τ_t .

It has to balance its budget **every** period. All investment is made at the household level and the law of motion is

$$k_{t+1} \leq x_t + (1 - \delta)k_t$$

- (a) (5pts) Define a TDCE for this economy, where the consumer faces a present value (Arrow-Debreu) budget constraint. Remember that the government balances its budget **every** period.
- (b) (10pts) Write down the system of equations that characterize the TDCE.
- (c) (5pts) Write down the planner's problem that the TDCE allocation solves. Remember that the SPP should not directly include prices or taxes.
- (d) (15pts) **Show** that the TDCE allocation of this economy solves this SPP.
- (e) (5pts) Suppose $\tau_{xt} = \tau_x \forall t$, $\tau_{ct} = \tau_c \forall t$, and $\tau_t = \tau \forall t$. Write down the Bellman equation for the corresponding planner's problem in part (c).

Extra Credit: Stochastic Competitive Equilibrium (5 points)

Consider a stochastic economy with one representative agent and one firm with a stochastic process s_t . Denote $s^t = (s_0, s_1, \dots, s_t)$ the entire sequence up to time t , where $Pr(s^t) = \pi(s^t)$ and assume that s_t follows a first-order Markov process.

The technology of the firm is given by $F(k_t, n_t, s_t) = A(s_t)k_t^\alpha n_t^{1-\alpha}$ (you can assume the production function is constant returns to scale) and preferences are given by a utility function $u(c_t)$. Consider the standard with depreciation rate δ and feasibility constraint where the sum of consumption and investment is equal to production. The agent is endowed with one unit of labor in each period.

Define an Arrow-Debreu Competitive Equilibrium in this setup (there are no taxes).